Published by Institute of Physics Publishing for SISSA/ISAS
Received: *November 16, 2004*
Accepted: *December 29, 2004*

Microscopic description of black rings in AdS/CFT

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Abstract: We discuss some aspects of the recently discovered BPS black ring solutions in terms of the AdS/CFT correspondence. In the type IIB frame in which the black ring carries the charges of the D1-D5-P system, we propose a microscopic description of the rings in the orbifold CFT governing this system. In our proposal, the CFT effectively splits into two parts: one part captures the supertube-like properties of the ring, and the other captures the entropy. We can also understand the black ring entropy by relating the geometry near the ring to BPS black holes in four dimensions, although this latter approach does not directly lead to an identification of black rings in terms of the D1-D5-P CFT.

Keywords: Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

An interesting new solution of five dimensional minimal supergravity was discovered recently [1]: a supersymmetric black ring with horizon topology $S^1 \times S^2$. More general solutions in eleven dimensional supergravity were subsequently obtained in [2, 3], as well as multi-ring solutions in [4, 5]. These solutions have their origins in prior studies of (non-BPS or singular) black rings [6]–[9], in studies of three-charge supertubes [10, 11], as well as in methods for classifying supersymmetric solutions [12, 13].

One of the most interesting features of these black rings is that, even after fixing all their conserved charges, they retain several discrete parameters which can be varied; furthermore, the area of the horizon depends on these parameters. Therefore, these black rings have "hair". This makes it especially interesting to see whether the entropy can be computed microscopically in string theory in terms of the AdS/CFT correspondence, and to see how the hair parameters are accounted for. In previous microscopic computations of black hole entropy following [14] one counts up all states with given values of conserved charges, but for the black rings one apparently needs to impose additional restrictions on the class of states to be counted.

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Mathur and collaborators [15]–[21] have initiated a program to argue that every microstate of the D1-D5-P system corresponds to a particular bulk solution (though not necessarily a smooth and classical solution of supergravity) with no event horizon. In this picture, the logarithm of the black hole entropy is equal to the total number of choices for the hair parameters. For some choices of parameters, the black ring solutions have vanishing entropy, and so can potentially serve as geometries dual to individual microstates in the manner envisaged by Mathur. One of our goals here is to identify these microstates. We will also see that despite having small curvature these supergravity solutions are singular, as they have cycles that shrink to zero size; whether or not the singularities can be resolved remains to be seen.

The black ring can be thought of as a supertube [22, 23], with three conserved charges and three dipole charges [10]. The dipole charges correspond to branes with one direction describing a topologically trivial closed curve in spacetime, and hence giving rise to no net charge. In the type IIB duality frame in which the black rings carries the conserved charges of the D1-D5-P system, the dipole charges are those of D1-branes, D5-branes, and Kaluza-Klein monopoles. Very near the ring the dipole branes effectively appear flat. Since the dipole branes are precisely those appearing in the construction of the four dimensional black hole [24, 25], it comes as no surprise that the black ring entropy is just a rewriting of the four dimensional black hole entropy, as we discuss in the next section.

On the other hand, after taking the near horizon decoupling limit, the black ring solutions are asymptotically $AdS_3 \times S^3 \times T^4$, and so should be describable in terms of the usual D1-D5 CFT. Most of what we know about this CFT comes from working at the point in moduli space in which it is free — the "orbifold point" (see, e.g., [26, 27] for reviews of the D1-D5 system and [28, 29] for more discussion of the moduli space.) Since the supergravity approximation is invalid at the orbifold point, direct comparisons between the two sides of the duality can only be made in certain cases. As we will see, the black ring solutions can be thought of as an amalgam of the two-charge supertube and the BMPV [30] black hole, and since we know that many of their properties are correctly captured at the orbifold point we can hope that the success carries over.

In particular, two-charge supertubes are described in terms of multiple "component strings" with angular momentum provided by fermion zero modes [15, 31], while the BMPV black hole is represented as a single component string with the angular momentum carried by fermion momentum modes [30]. With this in mind, we propose a microscopic description for the black rings in which the effective string of the D1-D5 system effectively splits into two parts, one corresponding to the supertube and one to the BMPV black hole. After making one phenomenological assumption about the length of the component strings, we are able to explain the entropy of all circular black rings. It will be interesting to see whether this assumption can eventually be derived from first principles.

Another way to view the microscopic description of the black rings is to regard the supergravity solutions as describing an RG flow. At the AdS_3 boundary one has the CFT of the D1-D5 system, which has $(4, 4)$ supersymmetry and $c_{UV} = 6N_{D1}N_{D5}$, while near the ring one has the CFT of the four-dimensional black hole, which is a $(4, 0)$ theory

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with $c_{IR} = 6n_{d1}n_{d5}n_{kk}$ (lower case letters denote the dipole charges). We will analyze the geometry at the IR end of the flow near the ring, but understanding the full flow from the boundary gauge theory point of view is left as an open question for the future.

Some of the observations we make below can also be found in [3], which appeared while this paper was being written.

2. The IIB Solution

The supersymmetric black ring was given an M-theory description in [2, 3] in terms of intersecting M2-branes and M5-branes. For AdS/CFT purposes it is more convenient to work in the duality frame in which the solution carries the same charges as the familiar D1-D5-P system. In this frame, the solution will also carry dipole "charges" of D1-branes, D5-branes, and KK-monopoles (this IIB solution also appears in [3], and its geometry was thoroughly investigated there)

To reach the D1-D5-P frame we compactify the M-theory solution along one of the M2 branes, and T-dualize three more times. We obtain a solution of type IIB supergravity compactified on T^5, where $x^{6,7,8,9}$ describe a T^4 of volume V_4, and x^5 is a circle of radius R_{KK}. The number of integral units of charge and their orientations are

$$N_1 \quad D1(5)\,, \qquad N_2 \quad D5(56789)\,, \qquad N_3 \quad P(5)\,. \tag{2.1}$$

The dipole branes have one common worldvolume direction curled up into a circle, which we will now denote as x (this is the ring direction of the black ring). The number and orientation of the dipole branes is then

$$n_1 \quad d5(x6789)\,, \qquad n_2 \quad d1(x)\,, \qquad n_3 \quad kk(x56789)\,, \tag{2.2}$$

where we are denoting dipole quantities by lower case letters. x^5 is the special KK circle of the kk-monopole.

The quantized charges above are related to the parameters appearing in the metric as

$$\begin{aligned} Q_1 &= \frac{(2\pi)^4 g\alpha'^3}{V_4} N_1\,, \qquad Q_2 = g\alpha' N_2\,, \qquad Q_3 = \frac{(2\pi)^4 g^2\alpha'^4}{V_4 R_{KK}^2} N_3\,, \\ q_1 &= \frac{g\alpha'}{R_{KK}} n_1\,, \qquad q_2 = \frac{(2\pi)^4 g\alpha'^3}{V_4 R_{KK}} n_2\,, \qquad q_3 = R_{KK} n_3\,. \end{aligned} \tag{2.3}$$

The charges N_i are measured as flux integrals at infinity. These are to be distinguished from the charges measured by flux integrals at the ring itself. The latter are not conserved in general, but do tell us more directly what the ring is made of. Denoting these by $\overline{N}_i$, they are given by (as will be seen from the explicit form of the solution)

$$\overline{N}_1 = N_1 - n_2 n_3\,, \qquad \overline{N}_2 = N_2 - n_1 n_3\,, \qquad \overline{N}_3 = N_3 - n_1 n_2\,. \tag{2.4}$$

We also have the corresponding definitions $\overline{Q}_1 = Q_1 - q_2 q_3$, etc. As one can see from the equations governing the black ring solutions [2], the difference between the asymptotic and near-ring charges comes from the charge carried by supergravity fields. This feature

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of the black ring solution is very similar to the Klebanov-Strassler solution [32], which also contains D-brane charges "dissolved" into fluxes via the Chern-Simons terms in the action.

The metric in string frame is

$$ds^2 = -(e_1)^2 + (e_5)^2 + \sqrt{Z_1 Z_2} ds_4^2 + \sqrt{\frac{Z_1}{Z_2}} ds_{T^4}^2 \,, \tag{2.5}$$

where

$$\begin{aligned} e_1 &= \frac{1}{Z_1^{1/4} Z_2^{1/4} Z_3^{1/2}} (dt + k_\psi d\psi + k_\phi d\phi) \\ e_5 &= -e_1 + \frac{Z_3^{1/2}}{Z_1^{1/4} Z_2^{1/4}} (dt + dx_5 - s_\psi d\psi - s_\phi d\phi) \,. \end{aligned} \tag{2.6}$$

The solution is easiest to express if we use a coordinate system [1] in which the metric of flat $\mathbb{R}^4$ is

$$\begin{aligned} ds_4^2 &= \frac{R^2}{(x-y)^2} \left[\frac{dy^2}{y^2-1} + (y^2-1) d\psi^2 + \frac{dx^2}{1-x^2} + (1-x^2) d\phi^2 \right] \\ &= d\rho^2 + \rho^2 (d\theta^2 + \sin^2\theta d\psi^2 + \cos^2\theta d\phi^2) \end{aligned} \tag{2.7}$$

related to the usual flat $\mathbb{R}^4$ coordinates by

$$\begin{aligned} \rho \sin\theta &= \frac{\sqrt{y^2-1}}{x-y} R, \qquad \rho\cos\theta = \frac{\sqrt{1-x^2}}{x-y} R \\ x &= -\frac{\rho^2 - R^2}{\Sigma}, \qquad y = -\frac{\rho^2 + R^2}{\Sigma} \end{aligned} \tag{2.8}$$

with

$$\Sigma = \sqrt{(\rho^2 - R^2)^2 + 4R^2 \rho^2 \cos^2\theta} \,. \tag{2.9}$$

The coordinate ranges are $-1 \le x \le 1$, $-\infty \le y \le -1$, and ψ and ϕ are 2π periodic.

In these coordinates, the Z_i functions (which would be harmonic in the absence of the dipole charges) are

$$\begin{aligned} Z_1 &= 1 + \frac{\overline{Q}_1}{2R^2}(x-y) - \frac{q_2 q_3}{4R^2}(x^2 - y^2) \\ &= 1 + \frac{\overline{Q}_1}{\Sigma} + \frac{q_2 q_3 \rho^2}{\Sigma^2} \,, \end{aligned} \tag{2.10}$$

with $Z_{2,3}$ given by cyclic permutations of the labels (123). The other quantities appearing in the solution are

$$\begin{aligned} k_\psi &= -\frac{1}{2}(q_1+q_2+q_3)(1+y) - \frac{1}{8R^2}(1-y^2) \left[\sum_i q_i \overline{Q}_i - q_1 q_2 q_3 (x+y) \right] \\ &= \frac{2(q_1+q_2+q_3) R^2 \rho^2 \sin^2\theta}{\Sigma(\rho^2 + R^2 + \Sigma)} + \frac{\rho^2 \sin^2\theta}{2\Sigma^2} \left[\sum_i q_i \overline{Q}_i + 2 q_1 q_2 q_3 \frac{\rho^2}{\Sigma} \right] \end{aligned}$$

$$
\begin{aligned}
k_\phi &= -\frac{1}{8R^2}(1-x^2)\left[\sum_i q_i\overline{Q}_i - q_1q_2q_3(x+y)\right] = -\frac{\rho^2\cos^2\theta}{2\Sigma^2}\left[\sum_i q_i\overline{Q}_i + 2q_1q_2q_3\frac{\rho^2}{\Sigma}\right] \\
s_\psi &= \frac{1}{2}q_3(1+y) = \frac{1}{2}q_3\frac{\Sigma-\rho^2-R^2}{\Sigma} \\
s_\phi &= -\frac{1}{2}q_3(1+x) = -\frac{1}{2}q_3\frac{\Sigma-\rho^2+R^2}{\Sigma}\,.
\end{aligned}
\tag{2.11}
$$

The dilaton is

$$
e^{-2\Phi} = \frac{Z_2}{Z_1}\,. \tag{2.12}
$$

The solution carries the angular momenta

$$
\begin{aligned}
J_\phi &= -\frac{R_{KK}V_4}{2(2\pi)^4\alpha'^4g^2}\left(\sum_i q_i\overline{Q}_i + 2q_1q_2q_3\right) = -\frac{1}{2}\sum_i n_i\overline{N}_i - n_1n_2n_3 \\
J_\psi &= -J_\phi + \frac{R_{KK}V_4}{(2\pi)^4\alpha'^4g^2}\left(q_1+q_2+q_3\right)R^2\,.
\end{aligned}
\tag{2.13}
$$

There is a black hole horizon located at $y=-\infty$, and the corresponding black hole entropy works out to be

$$
\begin{aligned}
S = \frac{A}{4G} = 2\pi\Big[&-\frac{1}{4}(n_1^2\overline{N}_1^2 + n_2^2\overline{N}_2^2 + n_3^2\overline{N}_3^2) + \frac{1}{2}(n_1n_2\overline{N}_1\overline{N}_2 + n_1n_3\overline{N}_1\overline{N}_3 + n_2n_3\overline{N}_2\overline{N}_3) - \\
&- n_1n_2n_3(J_\psi + J_\phi)\Big]^{1/2}\,.
\end{aligned}
\tag{2.14}
$$

One of our main goals is to make as much progress as we can in understanding the formula (2.14) from the point of view of the AdS/CFT correspondence. In the near horizon limit which we take below, the ring solution is seen to be asymptotically $AdS_3\times S^3\times T^4$, and so should be describable as a state (or ensemble of states) in the same boundary theory as the usual D1-D5-P system.

There is actually another, simpler, way to understand the entropy formula. Instead of taking the dipole branes to be curled into a circle we can take them to be flat, as was done in [11]. The resulting brane configuration has 6 spatial worldvolume directions, and if we wrap these on T^6 then we obtain a four dimensional black hole preserving 4 supercharges. The general entropy for this class of black holes is [33]

$$
S = 2\pi\sqrt{J_4}\,, \tag{2.15}
$$

where J_4 is the quartic $E_{7(7)}$ invariant, which can be expressed in the basis (x_{ij}, y^{ij}) as

$$
\begin{aligned}
J_4 = &-\frac{1}{4}(x_{12}y^{12} + x_{34}y^{34} + x_{56}y^{56} + x_{78}y^{78})^2 - (x_{12}x_{34}x_{56}x_{78} + y^{12}y^{34}y^{56}y^{78}) + \\
&+ x_{12}x_{34}y^{12}y^{34} + x_{12}x_{56}y^{12}y^{56} + x_{34}x_{56}y^{34}y^{56} + x_{12}x_{78}y^{12}y^{78} + \\
&+ x_{34}x_{78}y^{34}y^{78} + x_{56}x_{78}y^{56}y^{78}\,.
\end{aligned}
\tag{2.16}
$$

We note that (2.15) agrees with (2.14) under the identifications

$$
\begin{aligned}
x_{12} &= \overline{N}_1\,, \qquad x_{34} = \overline{N}_2\,, \qquad x_{56} = \overline{N}_3\,, \qquad x_{78} = 0\,, \\
y^{12} &= n_1\,, \qquad y^{34} = n_2\,, \qquad y^{56} = n_3\,, \qquad y^{78} = J_\psi + J_\phi\,.
\end{aligned}
\tag{2.17}
$$

These identifications can be read off by starting from the M-theory version of the near-ring solution as presented in [11, 2, 3], reducing to IIA along the momentum direction, and then comparing with [34, table 2]. We should also notice that x_{78}, which is zero in our case, corresponds to having a KK monopole whose special direction is along the ring. It will be interesting to see if the black ring solutions can be generalized to include this charge.

A microscopic computation of the entropy was given in [34, 35] following the approach of [36]. So in this sense, the entropy formula (2.14) is understood in terms of the theory living on the dipole branes. However, our main goal is to understand the entropy in terms of the D1-D5-P CFT, so that we can have a common understanding of the black ring and the usual D1-D5-P black hole.

3. Decoupling Limit

The black ring geometry has two physically interesting limits. One is obtained exactly like in the three charge black hole case, by removing the asymptotically flat region of the solution to obtain a near-horizon geometry dual to the D1-D5-P CFT. The other limit is obtained by zooming in further on the near-ring region. In this limit the metric simply becomes the flat ring metric.

We leave the analysis of the near-ring limit to section 6, and focus for now on the near-horizon decoupling limit. As usual, we wish to take $\alpha' \to 0$ while scaling coordinates and moduli such that the metric has an overall factor of α' (so that the action $S \sim \frac{1}{G}\int\sqrt{-g}R$ is finite in the limit). In the (x, y, ψ, ϕ) coordinates this is achieved by

$$V_4 \sim \alpha'^2 , \qquad R \sim \alpha' \tag{3.1}$$

with all other coordinates and moduli fixed. From (2.3) this implies the following scaling of the charges

$$Q_{1,2} \sim \alpha', \quad Q_3 \sim \alpha'^2, \quad q_{1,2} \sim \alpha', \quad q_3 \sim \alpha'^0 . \tag{3.2}$$

Note that $Q_1 \sim q_2 q_3$ (and permutations thereof) and so $\overline{Q}_i$ scale the same as Q_i. Inserting these scalings into the Z_i, we see that we should drop the 1 from $Z_{1,2}$ while retaining it for Z_3; this is the same as in the standard D1-D5-P case. Finally, since $q_{1,2}$ scale to zero compared to q_3, whenever we see the combination $(q_1 + q_2 + q_3)$, as in (2.11) and (2.13), we should replace it by q_3.

Upon taking the near-horizon limit, the large ρ behavior of the metric is

$$\begin{aligned} ds^2 \approx{}& \frac{\rho^2}{\sqrt{Q_1 Q_2}}(-dt^2 + dx_5^2) + \sqrt{Q_1 Q_2}\frac{1}{\rho^2}d\rho^2 + \sqrt{Q_1 Q_2}(d\theta^2 + \sin^2\theta d\psi^2 + \cos^2\theta d\phi^2) + \\ &+ \sqrt{\frac{Q_1}{Q_2}} ds_{T^4}^2 \end{aligned} \tag{3.3}$$

which is the usual $AdS_3 \times S^3 \times T^4$ with $\ell_{AdS} = (Q_1 Q_2)^{1/4}$. Therefore, the black ring should be describable in terms of the CFT of the D1-D5-P system.

We recall that the angular momentum on the S^3 becomes the SO(4) $\approx$ SU(2)$_L\times$SU(2)$_R$ R-charge symmetry of the CFT. Left and right moving fermions transform as doublets under the corresponding SU(2) factor. The diagonal SU(2) generators, normalized to have integer eigenvalues, are related to $J_{\psi,\phi}$ by

$$J_L = J_\psi - J_\phi, \quad J_R = J_\psi + J_\phi\,. \tag{3.4}$$

Henceforth, our discussion of the supergravity solutions will be strictly in the context of the near horizon limit.

4. Special cases

In order to identify the black ring in the CFT it is helpful to first review some simple, well understood, special cases.

4.1 D1-D5 $\rightarrow$ kk supertube

Set $Q_3 = q_1 = q_2 = 0$, leaving us with N_1 D1-branes, N_2 D5-branes and n_3 kk monopoles, the latter being a dipole charge. These solutions have been intensively studied [37]–[39] and [15, 31]. In order to avoid singularities or closed timelike curves we must set the ring radius to be

$$R = \frac{\sqrt{Q_1 Q_2}}{q_3}\,, \tag{4.1}$$

which gives

$$J_L = J_R = \frac{N_1 N_2}{n_3}\,. \tag{4.2}$$

These solutions are interpreted as being particular ground states in the Ramond sector of the CFT.

In particular, at the orbifold point in the moduli space, we can think of the CFT as being a string of length $2\pi N_1 N_2$, which can be broken up into "component strings", each of length 2π times an integer. On each component string live 4 real bosons, and 4 real right-moving and left-moving fermions. Two of the right-moving fermions carry $J_R = +1$, while the other two carry $J_R = -1$, and analogously for the left-movers. Upon quantizing the zero mode fermions we find that each component string carries integer charges with $|J_{L,R}| \leq 1$.

The state (4.2) corresponds to taking all component strings to have length $2\pi n_3$ and carrying $J_L = J_R = 1$, so that there are a total of $N_1 N_2/n_3$ components in total, carrying the net charge (4.2). The interpretation of more general states is spelled out in detail in [15, 38].

4.2 BMPV black hole

Setting $R = 0$ gives the BMPV black hole [30], which carries

$$J_L = \sum_i n_i \overline{N}_i + 2 n_1 n_2 n_3\,, \qquad J_R = 0\,. \tag{4.3}$$

The entropy of the black hole is

$$S = 2\pi \sqrt{N_1 N_2 N_3 - \frac{J_L^2}{4}}\,. \tag{4.4}$$

Note that this cannot be obtained directly from the entropy formula (2.14), since the $R \to 0$ limit is discontinuous in the geometry. Upon defining the new angular coordinates [40]

$$\begin{aligned}\tilde{\psi} &= \psi - \frac{J_L}{2N_1N_2R_{KK}}(t+x_5)\\ \tilde{\phi} &= \phi + \frac{J_L}{2N_1N_2R_{KK}}(t+x_5)\end{aligned} \tag{4.5}$$

the metric becomes

$$\begin{aligned}ds^2 &= \frac{\rho^2}{\sqrt{Q_1Q_2}}(-dt^2+dx_5^2) + \sqrt{Q_1Q_2}\frac{1}{\rho^2}d\rho^2 + \frac{Q_1Q_2Q_3 - C^2J_L^2/4}{(Q_1Q_2)^{3/2}}(dt+dx_5)^2 + \\ &+ \sqrt{Q_1Q_2}(d\theta^2 + \sin^2\theta d\tilde{\psi}^2 + \cos^2\theta d\tilde{\phi}^2) + \sqrt{\frac{Q_1}{Q_2}}ds^2_{T^4}\end{aligned} \tag{4.6}$$

with

$$C^2 = \left(\frac{(2\pi)^4 g^2\alpha'^4}{R_{KK}V_4}\right)^2 = \frac{Q_1Q_2Q_3}{N_1N_2N_3}\,. \tag{4.7}$$

The (ρ, t, x_5) part of the metric is that of an extremal, rotating BTZ black hole [41].

The entropy (4.4) is most readily obtained as follows. We have a $\mathcal{N} = (4,4)$ SCFT of central charge $c = 6N_1N_2$ and we wish to count the number of left-moving states in the Ramond sector with $L_0 = N_3$ and with the R-charge J_L. We can apply a spectral flow transformation to remove the R-charge. A spectral flow by η_L acts as

$$\begin{aligned}L_0 &\to L_0 + \eta_L J_L + \frac{c}{6}\eta_L^2\\ J_L &\to J_L + \frac{c}{3}\eta_L\,.\end{aligned} \tag{4.8}$$

If we take $\eta_L = -\frac{J_L}{2N_1N_2}$ then we get $J_L = 0$ and $L_0 = N_3 - \frac{J_L^2}{4N_1N_2}$. Cardy's formula, $S = 2\pi\sqrt{\frac{c}{6}L_0}$ then yields (4.4).

The coordinate transformation (4.4) is the bulk implementation of the spectral flow transformation [37, 38], as is evident from the transformed level appearing in the BTZ metric. We can also read off another important feature. In the new coordinates the periodicity $(x_5, \psi, \phi) \cong (x_5 + 2\pi R_{KK}, \psi, \phi)$ turns into

$$(x_5, \tilde{\psi}, \tilde{\phi}) \;\cong\; \left(x_5 + 2\pi R_{KK}, \tilde{\psi} - \frac{J_L}{2N_1N_2}2\pi, \tilde{\phi} + \frac{J_L}{2N_1N_2}2\pi\right). \tag{4.9}$$

Since $\frac{J_L}{N_1N_2}$ is not in general an integer, the effective length of the x_5 direction (i.e. the length appearing in the quantization condition for momenta in this direction) is N_1N_2 times its naive value.[1] So in terms of the component string picture of the (orbifold point) of the CFT, the BMPV black hole corresponds to having a single component string of length $2\pi N_1N_2$. Since there is only a single component string, the R-charge (or equivalently, the angular momentum on S^3) is carried by the momentum modes of the fermions, rather than just by the zero modes as for the supertube.

[1]The factor of $\frac{1}{2}$ which we ignored is due to the anti-periodicity of fermions.

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5. Black ring entropy

Examination of the structure of the general solution indicates that it combines features of the supertube and the BMPV black hole, and so it is natural to seek a CFT description which similarly combines their microscopic elements. To this end, we first separate the angular momenta into two parts

$$\begin{aligned}\text{Tube}: &\qquad J_{\text{tube}} = J_\psi + J_\phi = \frac{q_3 R^2}{C} \\ \text{BMPV}: &\qquad J_{\text{BMPV}} = J_\phi = -\frac{1}{2}\sum_i n_i \overline{N}_i - n_1 n_2 n_3\,,\end{aligned} \tag{5.1}$$

where C was defined in (4.7). Our basic proposal is that the effective string of length $2\pi N_1 N_2$ splits into two parts, which we call the tube string and the BMPV string, with lengths $2\pi L_{\text{tube}}$ and $2\pi L_{\text{BMPV}}$. The tube string is broken up into a number of component strings of equal length $2\pi\ell_c$, and carries J_{tube} in the fermion zero modes. The BMPV string, on the other hand, consists of a single component and carries the momentum, entropy, and J_{BMPV} as momentum excitations.

Since each component of the tube string carries $J_{\text{tube}} = 1$, we immediately find that the number of such components is

$$\frac{L_{\text{tube}}}{\ell_c} = J_{\text{tube}}\,. \tag{5.2}$$

Since we also have $L_{\text{tube}} + L_{\text{BMPV}} = N_1 N_2$, there is only one free parameter remaining, which we can take to be ℓ_c. A complete explanation would include a computation of ℓ_c from first principles; we will not be able to achieve this in general, but we will see how to understand the form of ℓ_c in limiting cases.

Given our interpretation, the entropy should take the BMPV form

$$S = 2\pi\sqrt{L_{\text{BMPV}} N_3 - J_{\text{BMPV}}^2}\,. \tag{5.3}$$

Next, in order to put the black ring entropy in a more suggestive form, we parameterize the angular momenta as

$$J_{\text{tube}} = \frac{\overline{N}_1 \overline{N}_2}{n_3} - \delta, \quad J_{\text{BMPV}} = -n_3 N_3 + \gamma\,, \tag{5.4}$$

which, after some algebra, brings the entropy (2.14) to the form

$$S = 2\pi\sqrt{n_1 n_2 n_3 \delta - \gamma^2}\,. \tag{5.5}$$

Another useful formula for γ is

$$\gamma = \frac{1}{2}(n_3 \overline{N}_3 - n_1 \overline{N}_1 - n_2 \overline{N}_2)\,. \tag{5.6}$$

We now examine some particular cases of increasing complexity.

5.1 Case 1: $\delta = \gamma = 0$

These are zero entropy solutions, and so should correspond to individual CFT microstates. In fact, these states are nicely understood by taking the length of the component strings to be

$$\ell_c = n_3 \tag{5.7}$$

which is the same length as for the pure supertube, as discussed in section 4. From (5.2) we have $L_{\rm tube} = \overline{N}_1 \overline{N}_2$, which implies

$$L_{\rm BMPV} = N_1 N_2 - \overline{N}_1 \overline{N}_2 = n_3^2 N_3\,. \tag{5.8}$$

We need to include fermionic excitations on the BMPV string in order to account for $J_{\rm BMPV}$. In order to get the maximally negative J we can fill up the Fermi sea with the negatively charged fermions. After a short calculation, one finds that the total charge one obtains in this way is[2]

$$J_\phi^{\rm max} = -\sqrt{L_{\rm BMPV} N_3}\,. \tag{5.9}$$

Given (5.8), we see that $J_\phi^{\rm max} = -n_3 N_3 = J_{\rm BMPV}$, and so the angular momentum is correctly accounted for.

Thus, these microstates nicely match up with the gravity solutions. Furthermore, they provide examples of geometries dual to microstates with nonzero D1, D5, and momentum charges. However, as we'll see in the next section, these geometries fail to cap off smoothly in the way that one needs in order to get a completely smooth supergravity description.

5.2 Case 2: $\delta \neq 0,\ \gamma = 0$

In this case the black ring has the nonzero entropy $S = 2\pi\sqrt{n_1 n_2 n_3 \delta}$. A first guess for how this comes about is as follows. To reduce $J_{\rm tube}$ we can start converting the tube component strings into the BMPV string. If each component string has length $\ell_c = n_3$ as above, then converting δ such components increases $L_{\rm BMPV}$ by $n_3\delta$. But from (5.3) this gives a microscopic entropy $S_{\rm micro} = 2\pi\sqrt{n_3 N_3 \delta} = 2\pi\sqrt{n_1 n_2 n_3 \delta + n_3 \overline{N}_3 \delta}$. We see that this is too large, although it does give the correct result in the regime $n_i \gg \overline{N}_i$.

In fact, there was no particularly good reason to assume that ℓ_c remains unchanged for nonzero δ. We now simply demand that ℓ_c change in such a way as to reproduce the correct entropy. Although this doesn't do much to explain the entropy in this case, it does have the virtue that the same formula for ℓ_c will continue to reproduce the entropy for nonzero γ, which is a nontrivial result.

In particular, if we again assume there are $J_{\rm tube}$ component tube strings, each of length ℓ_c, and that $L_{\rm BMPV} = N_1 N_2 - J_{\rm tube}\ell_c$, then we obtain the correct entropy by taking

$$\ell_c = \left(1 + \frac{\overline{N}_3}{N_3}\frac{\delta}{J_{\rm tube}}\right) n_3\,. \tag{5.10}$$

One can in principle hope to test this identification by performing the sort of scattering experiments in [15], but for now we leave it as a phenomenological assumption.

[2] This is just the usual bound on the BMPV angular momentum, as can be equivalently obtained, for example, from the spectral flow transformation discussed earlier.

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5.3 Case 3: $\delta \neq 0,\ \gamma \neq 0$

Proceeding as in Case 2 but now for arbitrary γ, and in particular using the *same* formula (5.10) for the component string lengths, we can work out the combination $L_{\rm BMPV}N_3 - J^2_{\rm BMPV}$ to find a nontrivial cancellation between terms linear in γ. Hence the two entropies (5.3) and (5.5) precisely match. Thus, the phenomenological assumption (5.10) correctly explains the entropy of all circular black rings.

We have now identified some additional zero entropy microstates corresponding to $\gamma^2 = n_1 n_2 n_3 \delta$. Again, they have zero entropy because the angular momentum bound is saturated, so the BMPV string is described by a filled Fermi sea. It is interesting to note that our supergravity solutions do not seem to be able to capture zero entropy microstates whose component string length is different from the value in (5.10). It would be nice to understand why it is so.

6. Near ring geometry

We now examine the geometry in the region near the ring in each of the three cases above. The main point is that the solutions approach $AdS_3 \times S^3/Z_{n_3} \times T^4$ near the ring, which is the same geometry as for a collection of D1-branes, D5-branes, and KK-monopoles; in other words, near the ring the dipole branes dominate (as is clear from the form of the Z_i for small Σ). Since one of the spatial Poincaré coordinates parallel to the boundary of AdS_3 is compactified, the geometries will all be singular whenever the area of the horizon shrinks to zero size. Thus, although we have been able to identify some low curvature geometries dual to microstates of the D1-D5-P system, they are not smooth supergravity solutions, since they do not cap off smoothly. On the other hand, it is then clear that the problem has been reduced to finding such smooth geometries for the $P = 0$ limit of the D1-D5-KK system; i.e. to finding smooth geometries corresponding to the ground states of the $4D$ black hole.

6.1 Case 1: $\delta = \gamma = 0$

If we write

$$\begin{aligned} \tilde{\psi} &= \psi - \frac{1}{q_3}(x_5 + t)\,, \qquad \tilde{\phi} = \phi + \frac{1}{q_3}(x_5 + t) \\ \tilde{x}_5 &= q_3 \psi - t\,, \qquad \cos(2\alpha) = x\,, \qquad \tilde{y} = -\sqrt{\frac{q_1 q_2 q_3^2}{2Q_1 Q_2}}\sqrt{y}\,, \end{aligned} \tag{6.1}$$

then the leading form of the metric for $\tilde{y} \to \infty$ is

$$ds^2 = \frac{\sqrt{q_1 q_2 q_3^2}}{\tilde{y}^2}(d\tilde{y}^2 - dt^2 + d\tilde{x}_5^2) + \sqrt{q_1 q_2 q_3^2}(d\alpha^2 + \sin^2\alpha\, d\tilde{\psi}^2 + \cos^2\alpha\, d\tilde{\phi}^2) + \sqrt{\frac{q_2}{q_1}} ds^2_{T^4}\,. \tag{6.2}$$

Note that we now have the angular identifications $(\tilde{\psi}, \tilde{\phi}) \cong (\tilde{\psi} - \frac{2\pi}{n_3}, \tilde{\phi} + \frac{2\pi}{n_3})$, giving rise to S^3/Z_{n_3}. We also have $\tilde{x}_5 \cong \tilde{x}_5 + 2\pi n_3 R_{KK}$. The geometry is singular at $\tilde{y} = \infty$ since the size of the $\tilde{x}_5$ circle shrinks to zero.

The spacetime central charge obtained from the Brown-Henneaux formula [42] is

$$c = \frac{3\ell_{\rm AdS}}{2G_3} = 6n_1n_2n_3\,. \tag{6.3}$$

To obtain this we took into account that G_3 is n_3 times larger than it would be for $AdS_3 \times S^3$, due to the reduced volume of S^3/Z_{n_3}.

This near ring geometry is the expected one arising from the flat limit of the d1, d5 and kk dipole branes (see [43] for discussion of string theory in this background, and [44] for discussion of the boundary CFT.) The corresponding conformal field theory has $(4,0)$ supersymmetry, and SU(2) R-symmetry. We can think of this theory as the IR fixed point of an RG flow starting from the $c = 6N_1N_2$, $(4,4)$, CFT at the original AdS boundary (the UV). Indeed, the IR central charge is strictly less than that in the UV: $n_1n_2n_3 < N_1N_2$. In the UV we started out in a nontrivial vacuum state which broke conformal invariance, much like starting at some point on the Coulomb branch. Some of the original degrees of freedom are massive in the new vacuum, and the remaining massless ones give rise to the CFT in the IR.

The definitions of $\tilde\psi$ and $\tilde\phi$ in the first line of (6.1) represent a spectral flow by $\eta_L = -\frac{1}{n_3}$, and maps the state to a state with $L_0 = \overline{L}_0 = c/24$ (same as for the Ramond vacua) and $J_R = -J_L = \frac{\overline{N}_1\overline{N}_2}{n_3}$. Furthermore, the fermions are periodic up to the phases $e^{2\pi i/n_3}$. This agrees with what one expects for one of the ground states of the tube string introduced earlier, consisting of $\frac{\overline{N}_1\overline{N}_2}{n_3}$ component strings, each of length $2\pi n_3$.

The redefinition $x_5 \to \tilde{x}_5$ corresponds to a more novel redefinition of the superconformal generators. It would be very interesting to understand from the CFT point of view why these new generators are the preferred ones in the IR; that is, to understand the RG flow better.

6.2 Case 2: $\delta \neq 0,\ \gamma = 0$

For nonzero δ, after performing the same coordinate transformation as in (6.1) we find the metric (6.2) plus the additional term

$$\frac{C\delta}{\sqrt{q_1q_2q_3^4}}(dt + d\tilde{x}_5)^2\,, \tag{6.4}$$

corresponding to an extremal BTZ black hole. In terms of the Virasoro algebra of this near ring geometry, the level is

$$L_0 = \delta\,, \tag{6.5}$$

which, combined with (6.3) and Cardy's formula $S = 2\pi\sqrt{\frac{c}{6}L_0}$ yields the entropy formula (5.5) for $\gamma = 0$.

6.3 Case 3: $\delta \neq 0,\ \gamma \neq 0$

We now write

$$\tilde\psi = \left(1 - \frac{\gamma}{n_1n_2n_3}\right)\psi - \frac{1}{q_3}(x_5 + t)\,, \qquad \tilde\phi = \phi + \frac{\gamma}{n_1n_2n_3}\psi + \frac{1}{q_3}(x_5 + t)$$

$$\tilde{x}_5 = q_3\psi - t\,, \qquad \cos(2\alpha) = x\,, \qquad \tilde{y} = -\sqrt{\frac{q_1q_2q_3^2}{2(Q_1Q_2 - q_3C\delta)}}\sqrt{y}\,, \tag{6.6}$$

to bring the leading form of the metric for $\tilde{y} \to \infty$ to

$$
\begin{aligned}
ds^2 = & \frac{\sqrt{q_1 q_2 q_3^2}}{\tilde{y}^2}(d\tilde{y}^2 - dt^2 + d\tilde{x}_5^2) + \frac{C^2}{(q_1 q_2 q_3^2)^{3/2}}(n_1 n_2 n_3 \delta - \gamma^2)(dt + d\tilde{x}_5)^2 + \\
& + \sqrt{q_1 q_2 q_3^2}(d\alpha^2 + \sin^2\alpha d\tilde{\psi}^2 + \cos^2\alpha d\tilde{\phi}^2) + \sqrt{\frac{q_2}{q_1}} ds_{T^4}^2 \, . \qquad (6.7)
\end{aligned}
$$

If $\frac{\gamma}{n_1 n_2}$ is an integer then the geometry is that of $AdS_3 \times S^3/Z_{n_3} \times T^4$, with the AdS_3 part being an extremal BTZ black hole, but more generally the metric is not a product. From (6.6) we see that there is an additional transformation of the angular coordinates on top of the spectral flow by $\eta_L = -\frac{1}{n_3}$. The meaning of this in terms of the original D1-D5 CFT is unclear to us.

7. Discussion

We have studied the black ring entropy from two points of view, corresponding to the asymptotic D1-D5 CFT and the near ring d1-d5-kk CFT. In the former case, which is the appropriate one if one wishes to achieve a common description of the black rings with the two-charge supertubes and the BMPV black hole, we proposed that the CFT effective string can be thought of as splitting into two parts which describe separate properties of the black rings. The general entropy formula in this description matches the black ring entropy after making a phenomenological assumption about the lengths of the component strings. Hopefully, a derivation of this assumption from first principles will be supplied in the future.

By studying the geometry near the ring, we saw the emergence of the d1-d5-kk system, and this lead to a simple understanding of the general entropy formula. This analysis also showed that the geometries dual to individual CFT microstates are singular, despite having small curvature. The possibility of obtaining a nonsingular solution via a more general supergravity Ansatz is left open.

There are a number of interesting aspects of these systems which deserve further study. Perhaps the most intriguing one is the relation between the length of the components of the CFT effective string and the parameters in the geometry. In section 5 we have argued that zero entropy geometries with nonzero γ and δ are dual to CFT microstates with component strings of length given by (5.10), and have used this phenomenological assumption to explain the entropy of all circular black rings. However, there are many similar CFT microstates whose component strings have a different length, yet their duals do not appear to be among the U(1) $\times$ U(1) invariant solutions we have considered. It is also unlikely that these microstates are dual to non-circular rings (a naive time of travel calculation suggests that the symmetry of the geometry does not change as one changes the length of the component strings). One possibility is that these represent multi-ring solutions [4, 5]. It would be very interesting to find the duals of these microstates, or, alternatively, explain why only the microstates (5.10) have geometric duals. It is also possible that these microstates simply do not exist when one takes into account the deformations away from the orbifold point of the CFT.

The U(1) × U(1) invariant black rings analyzed here appear to be a small subset of a much larger class of black rings of arbitrary shape and charge densities, parameterized by seven arbitrary functions [2]. The existence of this huge class of solutions is also supported by the Born-Infeld analysis of three charge supertubes of arbitrary shapes [10]. It is important to try to map out the microscopic description of these more generic geometries.

Another interesting avenue for future research is to better understand the RG flow between the two CFT descriptions discussed above. Since the near-ring metric only depends on the dipole charges and local branes densities of the black rings, but not on their shapes, one expects the end point of the RG flows to be universal, regardless of the shape of the ring. This very large class of RG flows to the common IR fixed point deserves a more thorough analysis.

Acknowledgments

We would like to thank Masaki Shigemori and Radu Roiban for discussions. The work of IB and PK is supported in part by the NSF grants PHY00-99590 and PHY01-40151.

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